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LAMINAR HEAT TRANSFER AND SKIN FRICTION ON A POROUS PLATE IN UNSTEADY MOTION

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Abstract—The compressible laminar boundary-layer flow over a semi-infinite isothermal flat plate in nearly quasi-steady motion with a special form of surface mass flux, $\rho_W v_W/\rho_\infty U(t) = \alpha\, Re^{-\frac{1}{2}}$, has been studied, and results of skin friction and heat transfer have been reported and discussed here with particular emphasis on effects of plate accelerations.

The results reveal that acceleration increases skin friction for both the case of suction and the case of blowing. For heat transfer, it is found that for a cold plate at high Mach numbers, acceleration produces a cooling effect on a suction plate and a heating effect on a blowing plate. On the contrary, for a hot plate at low Mach numbers, acceleration heats a suction plate and cools a blowing plate. For both skin friction and heat transfer the effect of flow unsteadiness is much stronger for flow with blowing than with suction.

NOMENCLATURE

C,	constant in the viscosity–temperature relationship, $\mu/\theta = C\,\mu_\infty/\theta_\infty$;
C_f,	dimensionless skin friction coefficient, $\tau_w/\frac{1}{2}\rho_\infty U^2$;
C_p,	specific heat of the gas at constant pressure, (a constant);
f,	dimensionless stream function, equation (4);
F	quasi-steady part of f, equation (5);
$f_0, f_1, \ldots$,	successive expansions of f, equation (5);
h,	temperature function, equations (9) and (10);
H,	quasi-steady part of h, equation (11.1);
$h_0, h_1, \ldots$,	successive expansions of h, equation (11.1);
k,	thermal conductivity of the gas;
Pr,	Prandtl number;
$\dot{q}$,	heat flux;
$Re(x, t)$,	Reynolds number, xU/ν;
r,	recovery factor;
s,	temperature function, equations (9) and (10);
S,	quasi-steady part of s, equation (11.2);
$s_0, s_1, \ldots$,	successive expansions of s, equation (11.2);
(x, y, t),	spatial and temporal coordinates, (fixed with respect to the plate);
(X, Y, T),	Howarth transformations of (x, y, t), see equation (1);
(u, v),	velocity vector corresponding to (x, y);
U,	plate velocity (in the negative x direction);
α,	a constant of order unity;
γ,	specific heats ratio of the gas (a constant);
$\xi_0, \xi_1, \xi_2, \ldots$,	parameters characterizing flow unsteadiness, $\xi_n = \dfrac{x^{n+1}}{U^{n+2}}\dfrac{d^{n+1}}{dt^{n+1}} U\ (t)$, $n = 0, 1 \ldots$;

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θ,	temperature;
Θ,	dimensionless temperature, equation (9);
μ,	dynamic viscosity;
ν,	kinematic viscosity;
ρ,	density;
σ,	similarity variable, $\frac{1}{2}Y(U/x\nu)^{\frac{1}{2}}$;
ψ,	stream function;
τ,	shear stress;
$\Delta\theta$,	prescribed temperature difference, $\theta_w - \theta_\infty$.

Subscripts

∞,	conditions of ambient gas;
w,	conditions at the surface of the plate;
aws,	adiabatic wall at quasi-steady condition.

1. INTRODUCTION

THIS paper reports a simplified study on the combined effects of flow unsteadiness and surface mass transfer on compressible laminar boundary-layer flows. It is believed that the results obtained bear some relevance to the important practical problem of the design of certain high-speed vehicles, e.g. missiles or rockets, which fly at continuously varying speeds.

A simple generalization of the Moore–Ostrach [1, 2] approach to unsteady boundary layers is made to allow for a specific surface mass distribution. A semi-infinite, isothermal, flat plate moving in its own plane with a continuously varying (otherwise arbitrary) velocity $U(t)$ is chosen for the study. The flow is assumed to be nearly quasi-steady, and the gas is assumed to be calorically perfect with a linear relationship between viscosity and temperature. The surface mass flux is of such a form that the requirement for the existence of similarity solutions in steady flow is met. Results presented here are based on a constant Prandtl number of 0·72. This work may serve to link the earlier work of Low [3] on steady similarity blowing to the works of Moore and Ostrach on unsteady boundary layers without mass transfer.

The analysis follows very closely that used by Moore and Ostrach. Only the results on skin friction and heat transfer will be reported here, and the readers are referred to Zien [4] or [1, 2] for detailed results and analysis.

2. ANALYSIS

A stream function, ψ, is introduced, following Moore[1], such that the unsteady continuity equation is satisfied. Then Howarth transformations

$$X = x, \qquad T = t, \qquad Y = \int_0^y \frac{\rho}{\rho_\infty}\,\mathrm{d}y \tag{1}$$

are used to reduce the equation of motion to incompressible form. We only note that the velocity components, (u, v), in the plate-fixed coordinates have the following expressions after transformations

$$u = \psi_Y \tag{2.1}$$

and

$$\rho v = \rho_w v_w + \rho_\infty \psi_X(X, 0, T) - \rho_\infty(\psi_X + uY_x + Y_t). \tag{2.2}$$

If the surface mass flux, $\rho_w v_w$, is put into the following form:

$$\rho_w v_w = -\rho_\infty \psi_x(X, 0, T) \tag{3}$$

then equation (2.2) reduces to

$$\rho v = -\rho_\infty(\psi_X + uY_x + Y_t).$$

For nearly quasi-steady flows, we write ψ as

$$\psi(X, Y, T) = [\nu X U(T)]^{\frac{1}{2}} f(\sigma; \xi_0, \xi_1, \xi_2, \ldots), \tag{4}$$

where σ is the Blasius variable and ξ's are parameters which form a diminishing sequence in nearly quasi-steady flows. Expanding f in terms of ξ's [1], we have

$$f = F(\sigma) + \xi_0 f_0(\sigma) + \xi_1 f_1(\sigma) + \xi_2 f_2(\sigma) + \ldots + \xi_0^2 f_{00}(\sigma) + \ldots \tag{5}$$

Successive ordinary differential equations for F, f_0, f_1, etc., can thus be obtained. The boundary condition, equation (3.1), now takes the following form:

$$-\frac{\rho_w v_w}{\rho_\infty U(T)} Re^{\frac{1}{2}}_{4} = \tfrac{1}{2}(F(0) + 3\xi_0 f_0(0) + 5\xi_1 f_1(0) + \ldots]. \qquad (6)$$

We consider here only the case in which

$$F(0) = -2\alpha = \text{const.} \qquad (7.1)$$

and

$$f_n(0) = 0, n = 0, 1, 2, \ldots \qquad (7.2)$$

Therefore we are restricted to a rather specific form of surface mass flux which varies with the plate speed, $U(T)$, as well as with X. Explicitly, it is required that

$$\frac{\rho_w v_w}{\rho_\infty U} = \alpha\left(\frac{\nu}{UX}\right)^{\frac{1}{2}}. \qquad (8)$$

We remark here that the resulting systems of differential equations for F and f's, except for boundary condition $F(0)$, are exactly the same as those given by Moore [1]. The effect of surface mass flux enter into the system only through the boundary condition, equation (7.1).

Next we consider the temperature field which is decoupled from velocity field by the assumption of linear variation of viscosity with temperature. The treatment of the energy equation which governs the temperature field is entirely analogous to that used by Ostrach [3]. We shall only outline briefly the procedure here.

First, a dimensionless temperature, Θ, defined by

$$\Theta \equiv \frac{\theta - \theta_\infty}{\theta_w - \theta_\infty}. \qquad (9)$$

is introduced, and we write

$$\Theta(\sigma, T; \xi_n) = h(\sigma; \xi_n) + \frac{U^2(T)}{2Cp(\theta_w - \theta_\infty)} s(\sigma; \xi_n). \qquad (10)$$

Next, h and s are expanded in terms of ξ's. Thus,

$$h = H(\sigma) + \xi_0 h_0(\sigma) + \xi_1 h_1(\sigma) + \ldots + \xi_0^2 h_{00}(\sigma) + \ldots \qquad (11.1)$$

$$s = S(\sigma) + \xi_0 s_0(\sigma) + \xi_1 s_1(\sigma) + \ldots + \xi_0^2 s_{00}(\sigma) + \ldots \qquad (11.2)$$

Finally systems of equations for H, h_0, h_1, S, s_0, s_1, etc., are obtained. They are identical in form to those of Ostrach [2], including all the boundary conditions. The effect of surface mass flux enters implicitly into the temperature distribution only through the velocity functions F, f_0, f_1, etc., which now appear as coefficients of the successive differential equations for the temperature.

The nine systems of ordinary differential equations for F, f_0, f_1, H, h_0, h_1, S, s_0 and s_1 are then solved numerically on the Univac 1108 computer at the Case Western Reserve University, for α ranging from -10 to 0·6. Since the boundary layer is near blow-off as α gets close to α_c (=0·619, see [5]), difficulties with regard to the convergence of F begin to manifest for α close to α_c. Higher-order solutions with $\alpha > 0{\cdot}5$ contain certain nonuniformity, and therefore have only limited physical significance.

3. RESULTS AND DISCUSSION

Complete numerical results of various profiles to order ξ_1 can be found in Zien [4], and we shall only discuss the results of skin friction and heat transfer here.

3.1 *Skin friction*

The skin friction coefficient, C_f, is given by

$$C_f = \tfrac{1}{2} Re^{-\frac{1}{2}}[F''(0) + \xi_0 f_0''(0) + \xi_1 f_1''(0) + \ldots]. \qquad (12)$$

Figure 1 illustrates the behavior of skin friction as a function of α. The quasi-steady part, $F''(0)$, needs no comment, and we only note that the result reduces correctly to that of the asymptotic suction case for large suctions. The contribution of plate acceleration is represented by f_0''. It is positive for all α and its magnitude

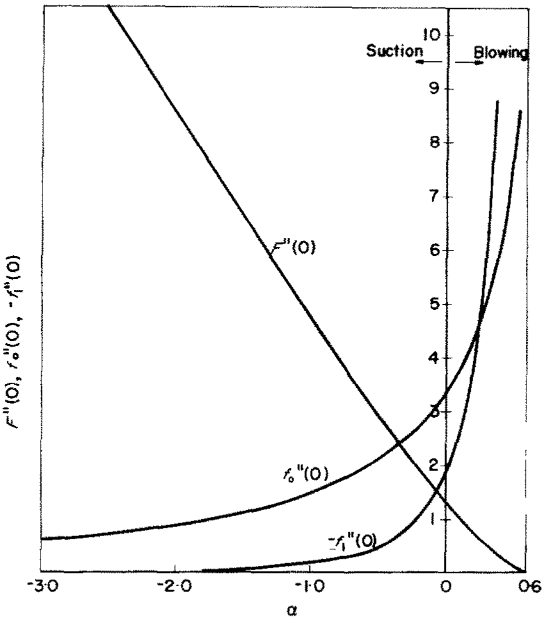

FIG. 1. Quantities related to skin friction [equation (12)].

increases with increasing α, indicating that plate accelerations tend to increase instantaneous local skin friction, and that the effect is more pronounced in the blowing case than in the suction case. The contribution of ξ_1, $\sim f_1''(0)$, is negative for all α, and its magnitude also increases with increasing α.

3.2 *Heat transfer*

The instantaneous heat flux from the fluid to the plate, $-\dot{q}_w$, is given by

$$-\dot{q}_w = \frac{1}{2}\frac{1}{P_r}(C_p\theta_\infty)(C\rho_\infty\mu_\infty U/X)^{\frac{1}{2}}\left\{\frac{\Delta\theta}{\theta_\infty}[H'(0) + \xi_0 h_0'(0) + \xi_1 h_1'(0) + \dots] + \frac{\gamma-1}{2}M_\infty^2[S'(0) + \xi_0 s_0' + \xi_1 s_1'(0)\dots]\right\}. \quad (13)$$

Quantities related to heat transfer are shown in Figs. 2 and 3 as functions of α.

The results for the case of a hot plate with low M_∞ will first be discussed (Fig. 2). Here the contribution to heat flux comes mainly from the static temperature difference, $\Delta\theta/\theta_\infty$. The quasi-steady results, $\sim H'(0)$, indicate that heat flows from the plate to the fluid for both suction and blowing, and that its magnitude decreases from suction to blowing. The effect of acceleration, $\sim h_0'(0)$, is to heat a suction plate and to cool a blowing plate. The behaviour of $h_1'(0)$ which relates to the effect of the rate of change of the plate acceleration, ξ_1, is seen to be the opposite.

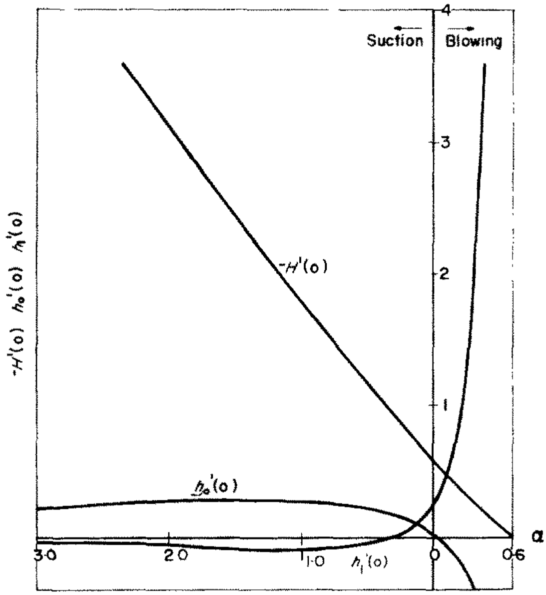

FIG. 2. Quantities related to heat transfer for $\Delta\theta/\theta_\infty \gg (\gamma - 1)M_\infty^2$.

Figure 3 illustrates the heat transfer for the case of high Mach number flight of a relatively cold plate ($M_\infty^2 \gg \Delta\theta/\theta_\infty$). The quasi-steady result, $\sim S'(0)$, reveals that heat flows from the fluid to the plate for both suction and blowing, and that its magnitude decreases from suction to blowing. Plate acceleration here has the effect of cooling a suction plate and heating a blowing plate, opposite to the previous case.

3.3 *Quasi-steady recovery factor*

For plates flying at high speeds, the quasi-steady approximation is usually adequate. The

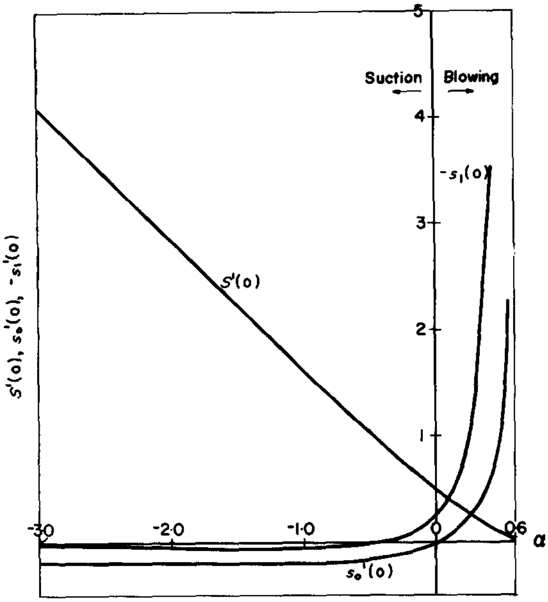

FIG. 3. Quantities related to heat transfer for $\Delta\theta/\theta_\infty \ll (\gamma - 1)M_\infty^2$.

dependence of the adiabatic wall temperature, θ_{aws} and also the recovery factor, r_s, on the parameter α can easily be determined in this limit. Let $\dot{q}_w = 0$ in equation (13). θ_{aws} is immediately determined as

$$\frac{\theta_{aws}}{\theta_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2 r_s(\alpha), \tag{14}$$

with

$$r_s(\alpha) = -\frac{S'(0;\alpha)}{H'(0;\alpha)}. \tag{15}$$

Figure 4 gives the result of $r_s(\alpha)$. It should be pointed out that the results of r_s due to Low [3] for $\alpha > 0$ are reproduced within 0·1 per cent,

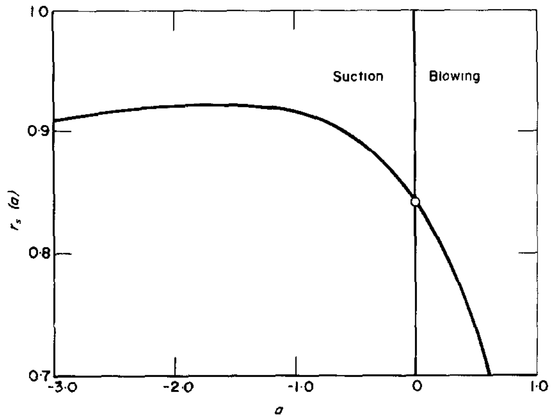

FIG. 4. Recovery factor for quasi-steady flows.

and that those for $\alpha < 0$ are believed to be new. Note that r_s has a mild maximum in the interval $-2{\cdot}0 < \alpha < -1{\cdot}0$.

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TRANSFERT DE CHALEUR LAMINAIRE ET FROTTEMENT SUPERFICIEL SUR UNE PLAQUE POREUSE EN MOUVEMENT INSTATIONNAIRE

Résumé—On étudie l'écoulement à couche limite laminaire compressible sur une plaque plane isotherme semi-infinie en mouvement quasi-stationnaire dans le cas d'un flux massique pariétal de la forme particulière $\rho_w v_w/\rho_\infty\, U(t) = \alpha R^{3-\frac{1}{4}}$. Tout en considérant en particulier les effets d'accélération de la plaque, on donne et discute les résultats concernant le frottement superficiel et le transfert de chaleur. Les résultats révèlent que, dans les cas de succion comme dans ceux de souflage, l'accélération augmente le frottement superficiel. Si l'on considère le transfert de chaleur, on remarque que, pour une plaque froide à des nombres de Mach élevés, l'accélération produit un effet de refroidissement sur une plaque avec succion et un effet d'échauffement sur une plaque avec soufflage. Au contraire, pour une plaque chaude à des nombres de

Mach bas, l'accélération échauffe une plaque avec succion et refroidit une plaque avec soufflage. Pour le frottement superficiel et le transfert de chaleur, l'effet d'instabilité de l'écoulement est bien plus important avec soufflage qu'avec succion.

LAMINARE WÄRMEÜBERTRAGUNG UND WANDREIBUNG AN EINER PORÖSEN PLATTE BEI INSTATIONÄRER BEWEGUNG

Zusammenfassung—Es wird die kompressible laminare Grenzschichtströmung über eine halbunendliche isotherme ebene Platte bei nahezu quasistationärer Bewegung mit einer speziellen Form des Oberflächen-Massenstroms, $\varphi_w V_w/\varphi_\infty U(t) = Re^{-\frac{1}{2}}$ untersucht. Ergebnisse für die Wandreibung und den Wärmeübergang werden gebracht und hier unter besonderer Betonung der Auswirkungen von Plattenbeschleunigungen erörtert.

Die Ergebnisse zeigen, dass Beschleunigung sowohl bei Absaugen wie bei Ausblasen die Wandreibung erhöht. Für die Wärmeübertragung findet man, dass bei kalter Platte und hohen Mach-Zahlen Beschleunigung bei Absaugen die Platte kühlt und bei Ausblasen heizt. Im Gegensatz dazu wird bei heisser Platte und niedrigen Mach-Zahlen durch Beschleunigung bei Absaugen die Platte geheizt und bei Ausblasen gekühlt. Sowohl für die Wandreibung wie für die Wärmeübertragung ist der Einfluss der instationären Bewegung viel stärker beim Ausblasen als beim Absaugen.

ЛАМИНАРНЫЙ ТЕПЛООБМЕН И ПОВЕРХНОСТНОЕ ТРЕНИЕ НА ПОРИСТОЙ ПЛАСТИНЕ ПРИ НЕУСТАНОВИВШЕМСЯ ДВИЖЕНИИ

Аннотация—Было рассмотрено течение в сжимаемом ламинарном пограничном слое на полуограниченной плоской изотермической плсатине при почти квазистационарном движении с определенным массовым расходом на поверхности, $\rho_W V_W/\rho_\infty U(t) = \alpha R^{1/2}$; приведены результаты вычислений поверхностного трения и теплообмена, которые обсуждаются относительно влияния ускорения на пластине.

Результаты показали, что ускорение увеличивает поверхностое трение как при отсасывании, так и в случае вдува. Что касается теплообмена показано, что для холодной пластины при больших числах Маха ускорение вызывает охлаждение на пластине с отсосом и нагревание на пластине с вдувом. Наоборот, для горячей пластины при малых числах Маха, ускорение нагревает пластину с отсосом и охлаждает пластину с вдувом. Как для поверхностного трения, так и для теплообмена нестационарность потока сказывается значительно сильнее для течения с вдувом, чем с отсосом.